

Wireless Embedded Systems

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Course Overview

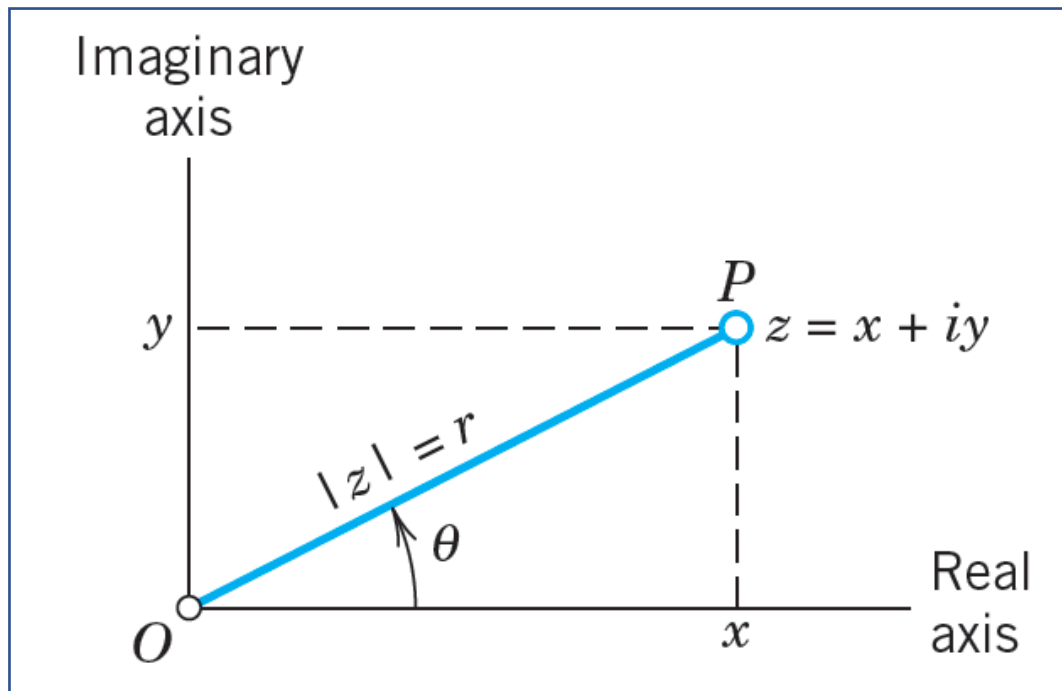
- Tentative Schedule

#	Topic	Lab	Assignment
1	Introduction		
2	Review: Python Basics, Signal & Systems	#1, 2	#1
3			
4	Digital Signals and Sampling	#3	
5			
6	Analog & Digital Modulation		#2
7			
8	Midterm Exam		
9	Software-Defined Radio	Env. Setup	#3
10			
11	Wireless Communication Experiments (Spectrum Monitoring, Communication)	#4 #5 #6	
12			
13			
14			
15	Final Exam		

Complex Numbers

- A number that describes two number!

$$z = x + iy = re^{i\theta}$$



Complex Numbers

- A number that describes two number!

$$z = x + iy = re^{i\theta}$$

- Addition & Subtraction => Cartesian form
- Multiplication & Division => Polar form (phasor)

	Multiplication	Division
Absolute Value (Modulus)	$ z_1 z_2 = z_1 z_2 $	$\left \frac{z_1}{z_2} \right = \frac{ z_1 }{ z_2 }$
Argument	$\arg(z_1 z_2) = \arg z_1 + \arg z_2$	$\arg(z_1 / z_2) = \arg z_1 - \arg z_2$

Complex Numbers

- Prove following De Moivre's Formula

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin \theta$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$e^{i\theta} + 1 = 0$$

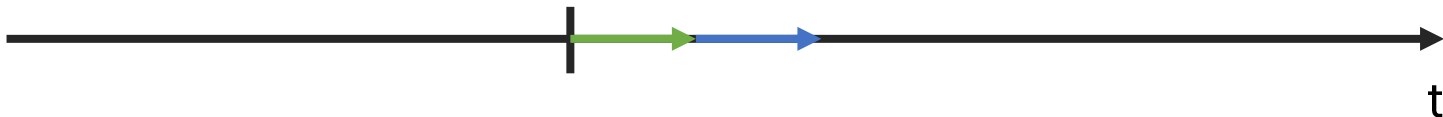
Complex Numbers

- Euler's Identity : Derivative Equation. (미분 방정식)

$$e^{i\theta} + 1 = 0$$

- Consider $\frac{d}{dt} e^t = e^t$
(속도) (위치)

- Multiplication of $i \Rightarrow$ Rotation



Complex Numbers

- Euler's Identity : Taylor Series

$$e^{i\theta} + 1 = 0$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

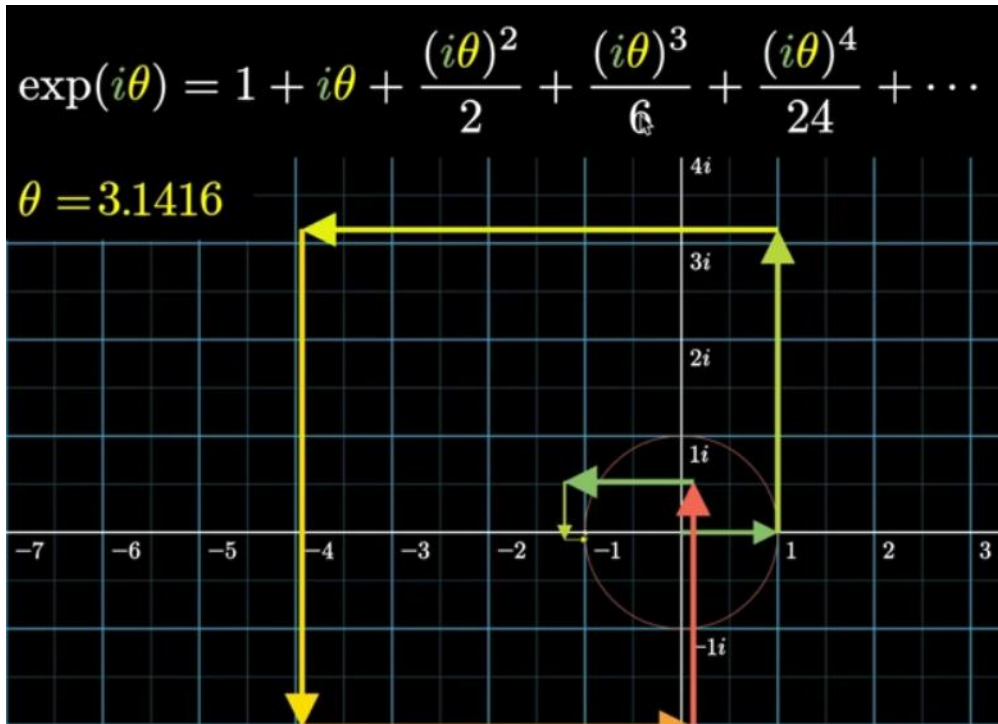
$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \quad \text{for all } x$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \quad \text{for all } x$$

Complex Numbers

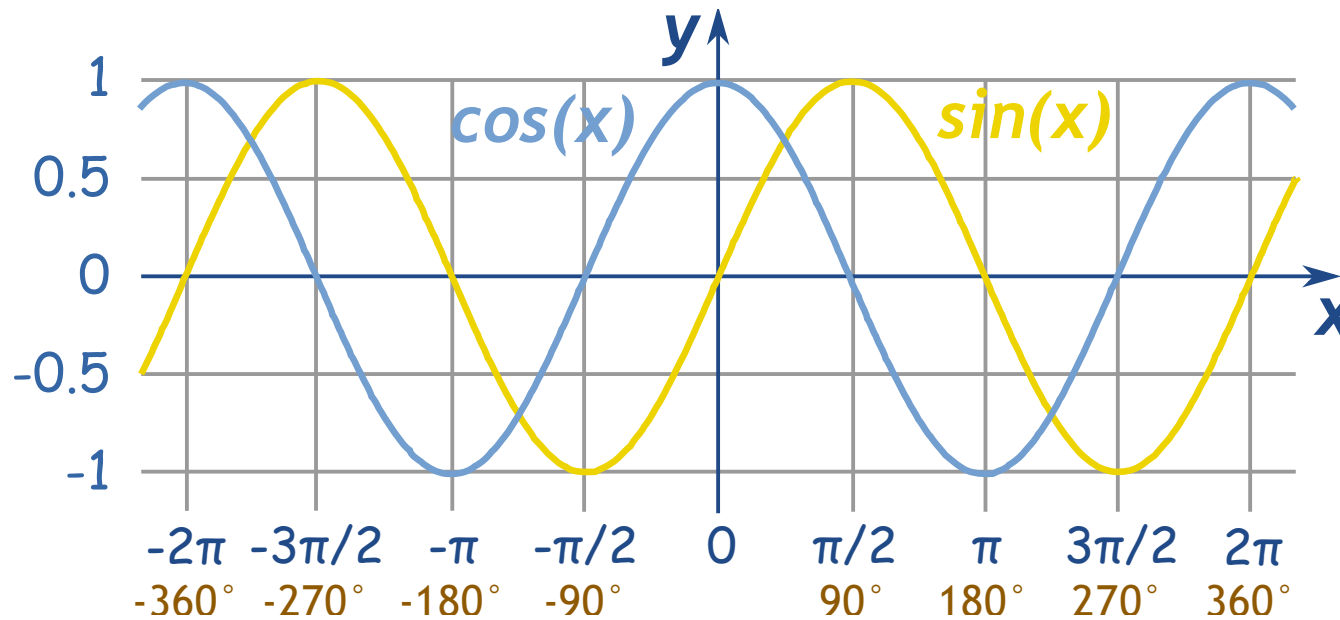
- Euler's Identity : Taylor Series in Complex Plane

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$



Complex Numbers

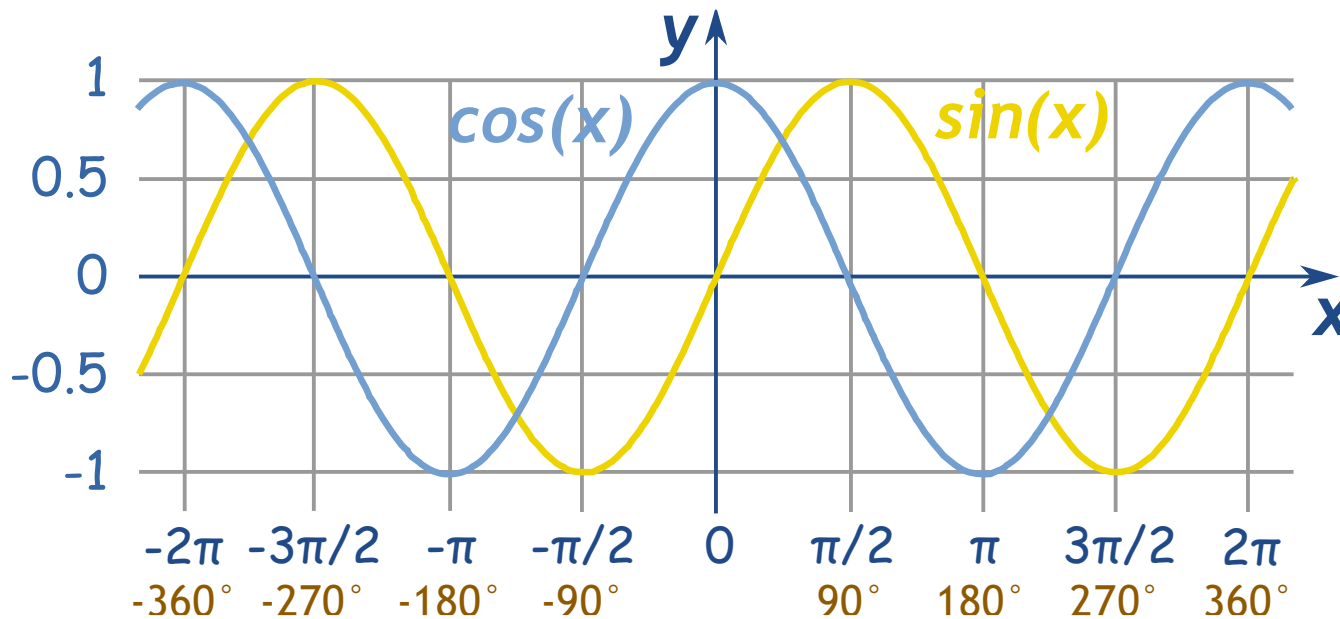
- A sinusoidal signal
 - Amplitude [voltage]
 - Phase [degree, radian]
 - Frequency [Hz]



Complex Numbers

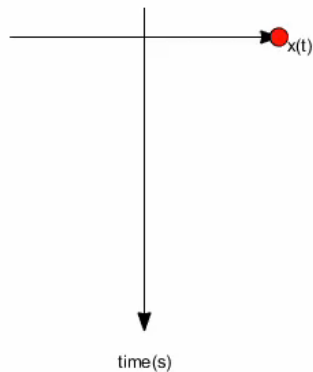
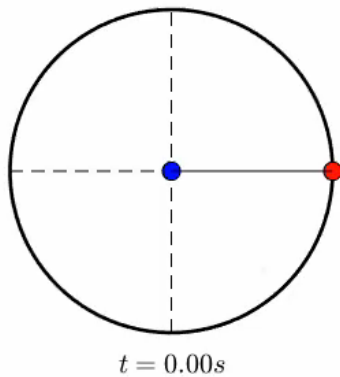
- Think about the **orthogonality**

$$\int \sin(kx) \cdot \cos(nx) dx = 0$$



Complex Numbers

- Euler's Identity
 - Orthogonality of sinusoidal & complex number.



$$\omega = 2\pi f$$

$$e^{j\omega t} = \cos \omega t + j \sin \omega t$$

Complex Numbers

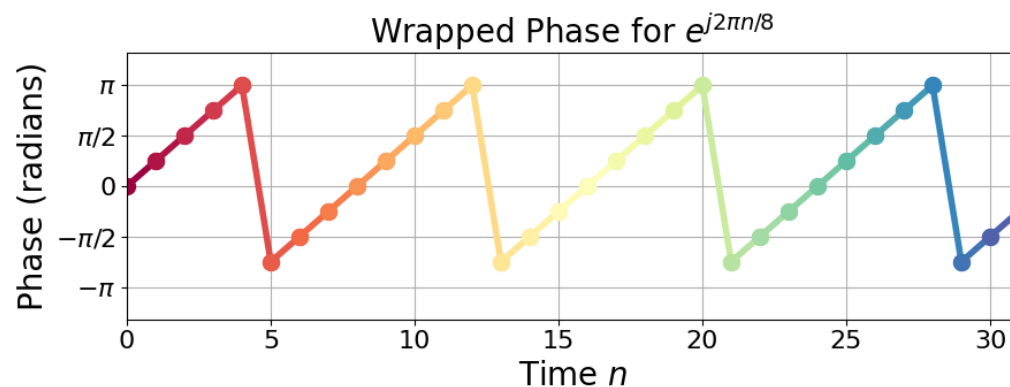
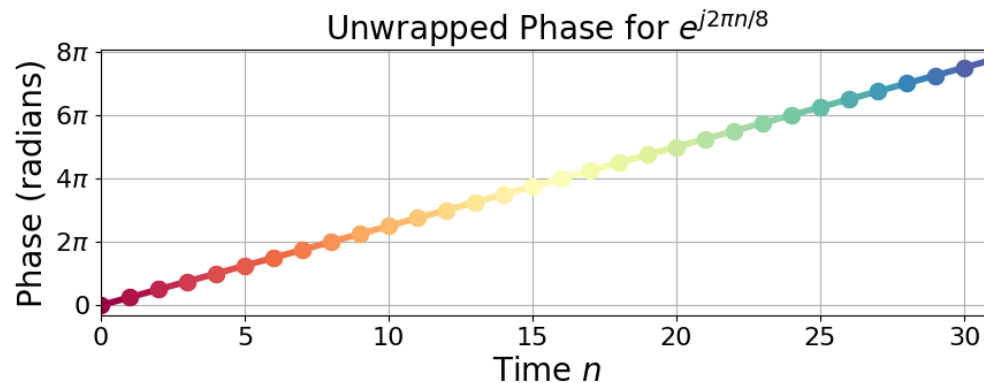
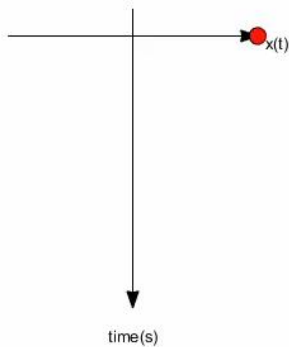
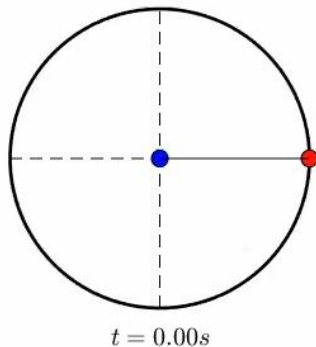
- Euler's Identity
 - Is **complex** exponential also orthogonal?

$$e^{j\omega t} = \cos \omega t + j \sin \omega t$$

$$\int e^{j\omega_1 t} \cdot e^{j\omega_2 t} dx = 0?$$

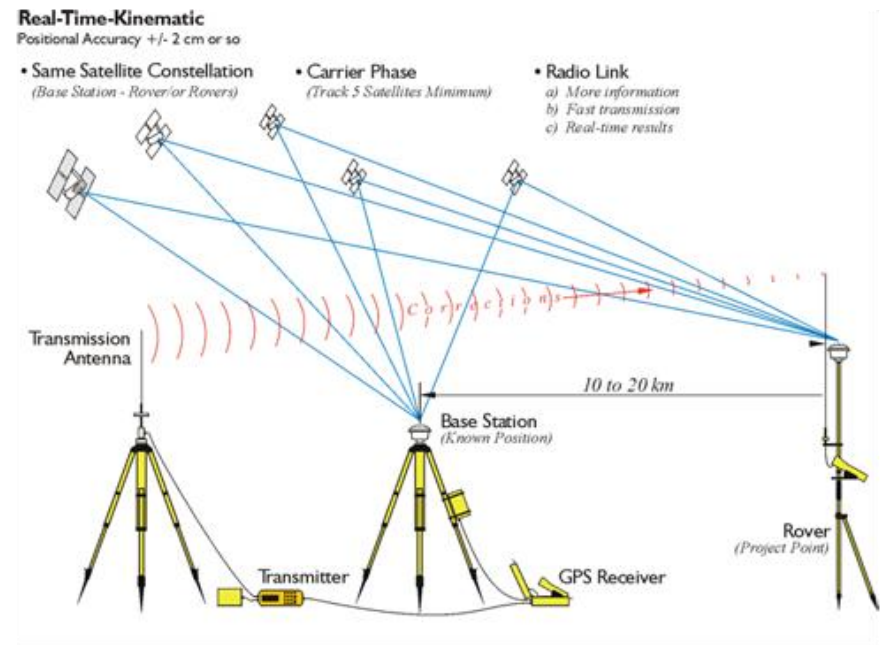
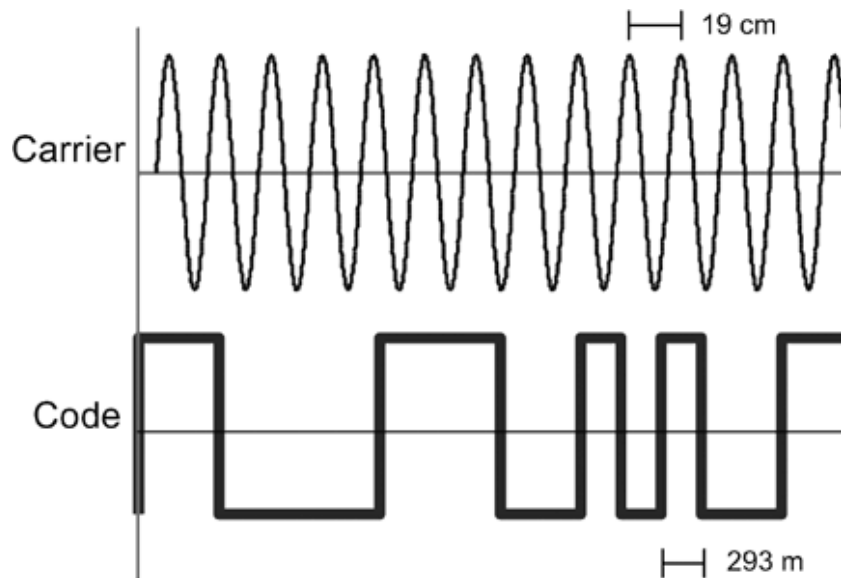
Complex Numbers

- The phase ambiguity
 - $y = \sin(\omega t)$



Complex Numbers

- Why phase is important?
 - GPS L1 Carrier Frequency : 1575.42 MHz ($\lambda = 19\text{cm}$)
 - GPS L1 C/A Code Rate : 1.023MHz ($\lambda = 293\text{m}$)
 - The light of speed, wavelength, and time



Complex Numbers

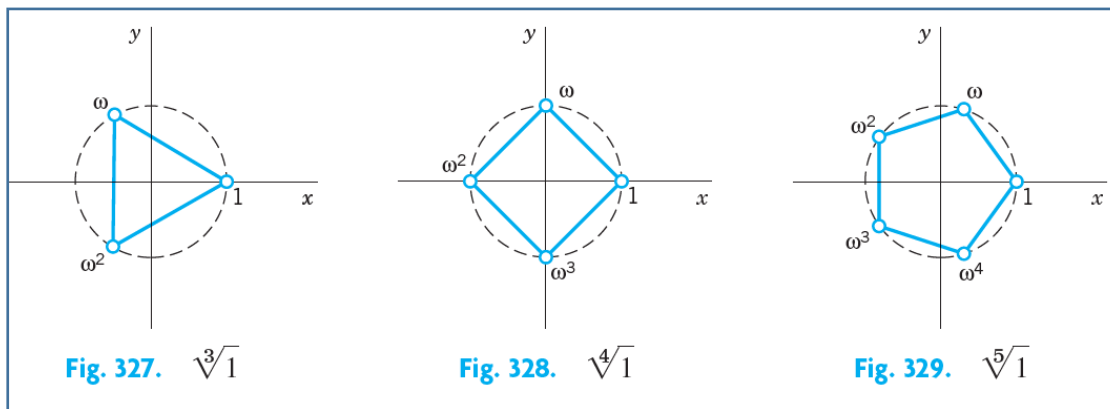
- Roots of Complex number

- In shortly,

$$\sqrt[n]{z} = \sqrt[n]{r} \left(\cos \frac{\theta + 2k\pi}{n} + i \sin \frac{\theta + 2k\pi}{n} \right)$$

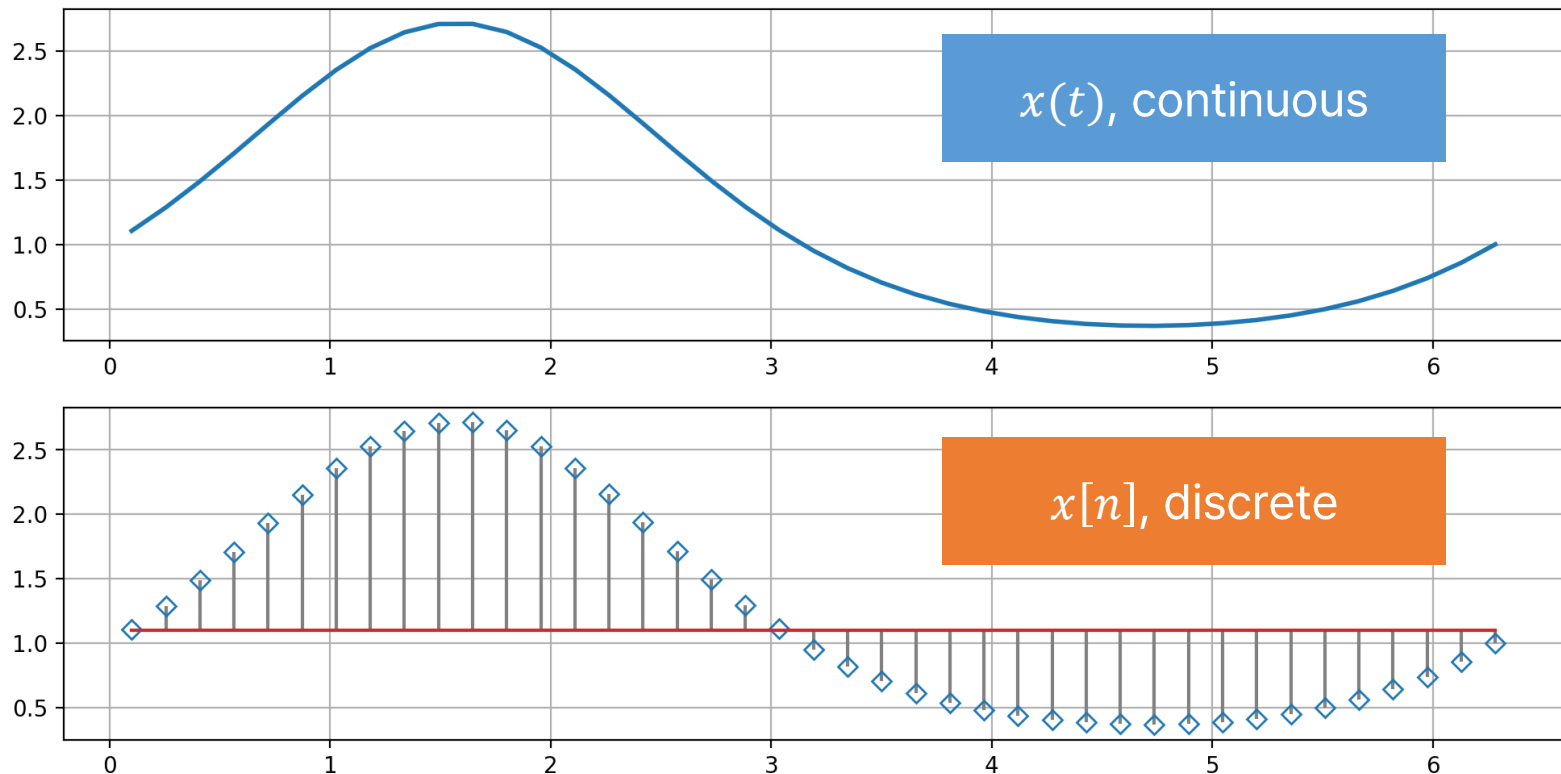
- And in case of $z = r = 1, (\arg z = 0)$

$$\sqrt[n]{1} = \cos \frac{2k\pi}{n} + i \sin \frac{2k\pi}{n}$$



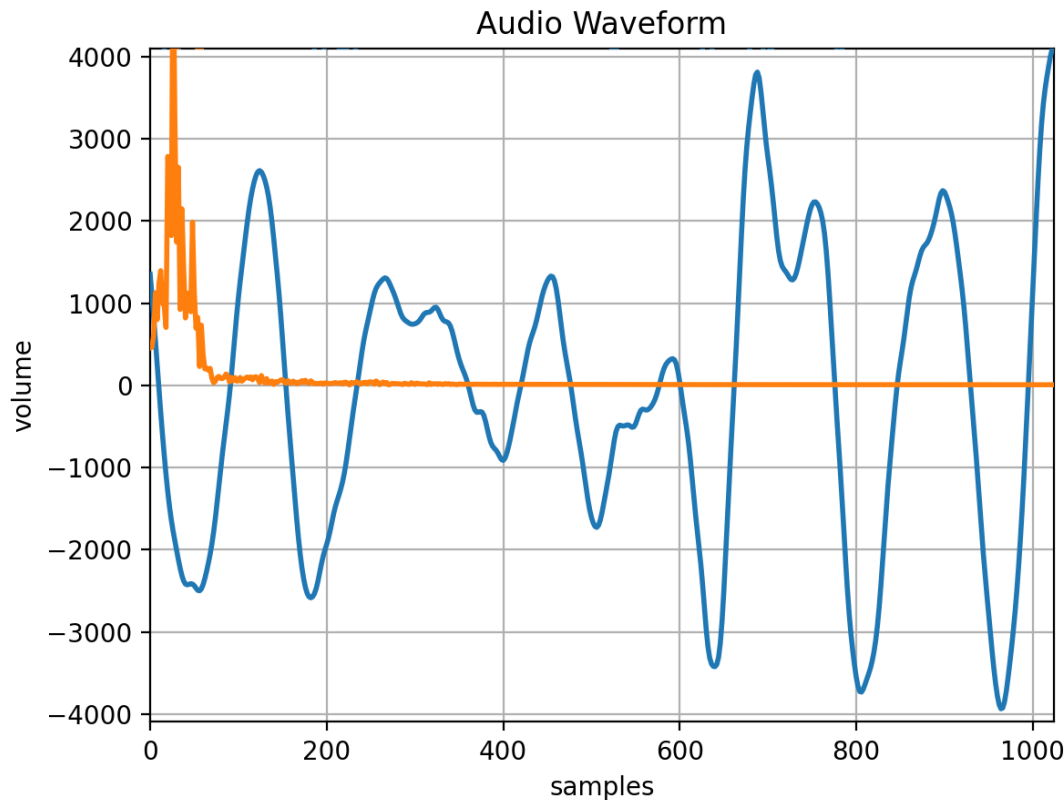
Review: Signals and Systems

- Signal
 - Physical quantity
 - Time domain function



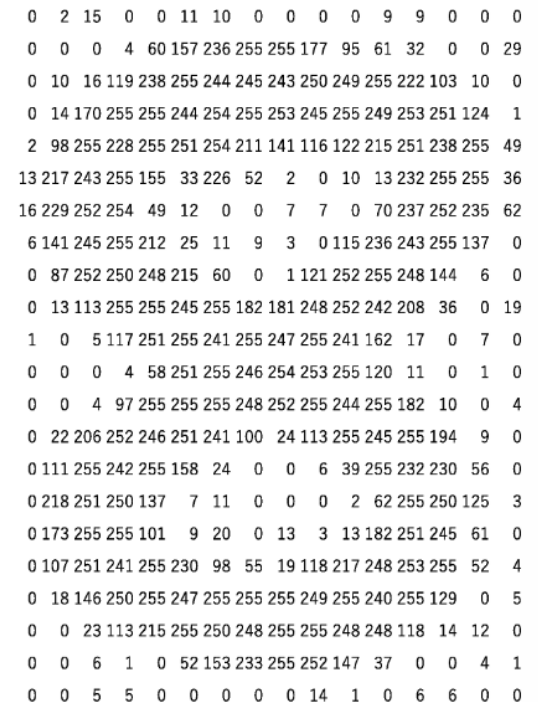
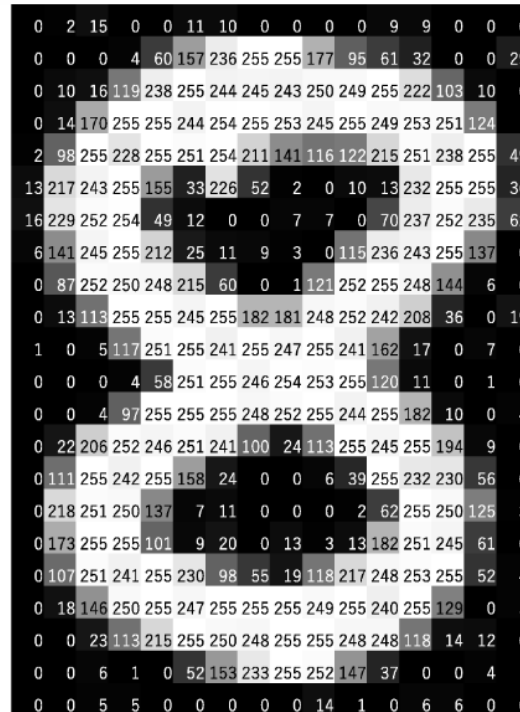
Review: Signals and Systems

- What are Signals?
 - Demo: Python Audio Waveform and its Frequency



Review: Signals and Systems

- Signals can be in higher dimension
 - Image : 3D data (x,y,value)
 - Video : 4D data (x,y,t,value)

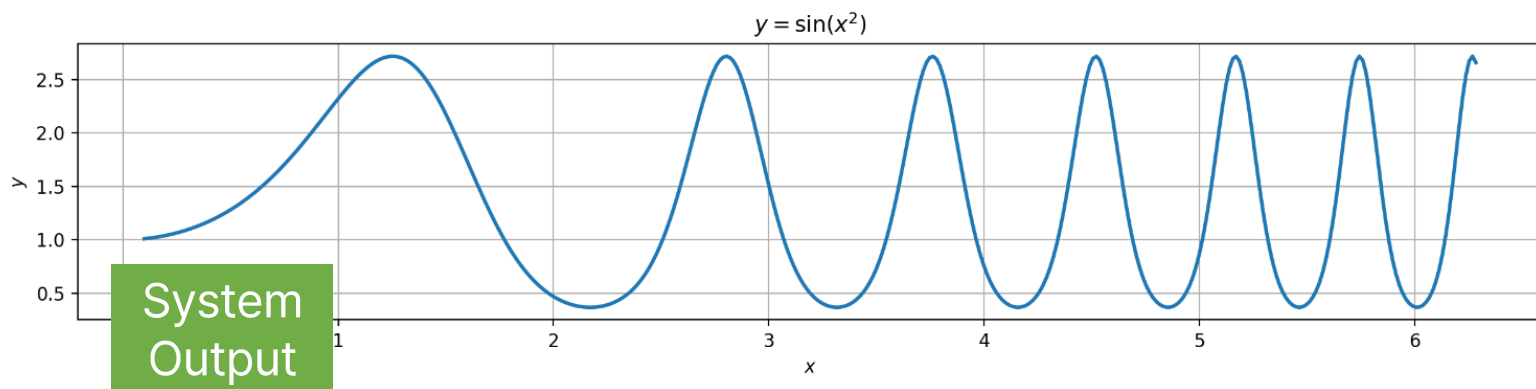
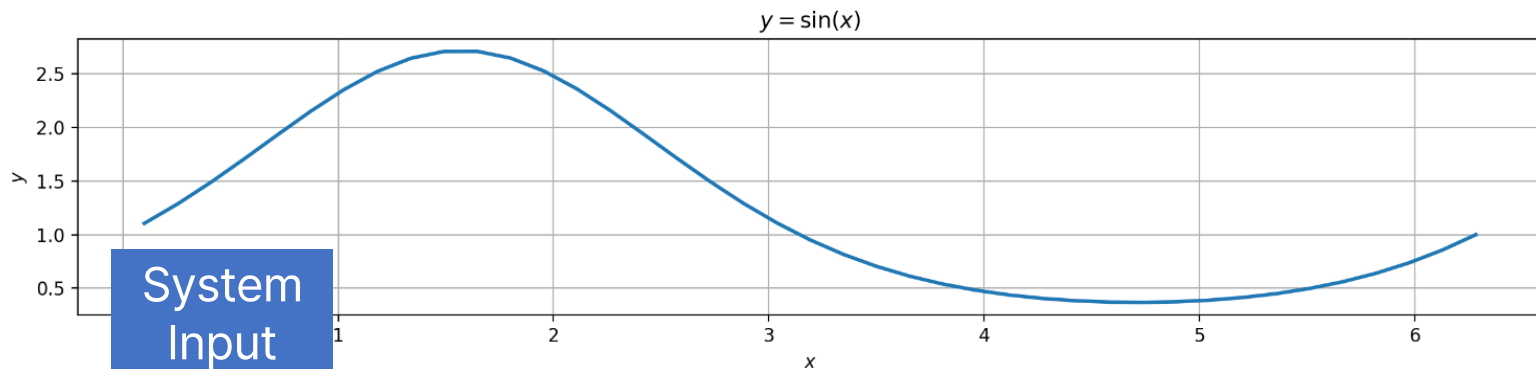
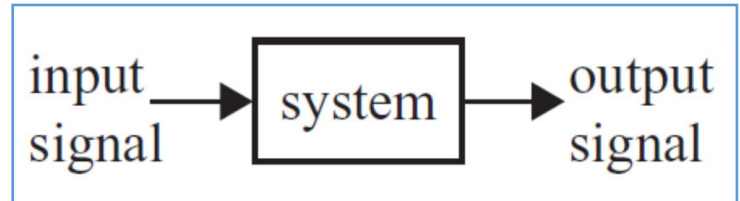


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Review: Signals and Systems

- What is a System?

- A mapping, **a function to a function**.
- What is system for below example?

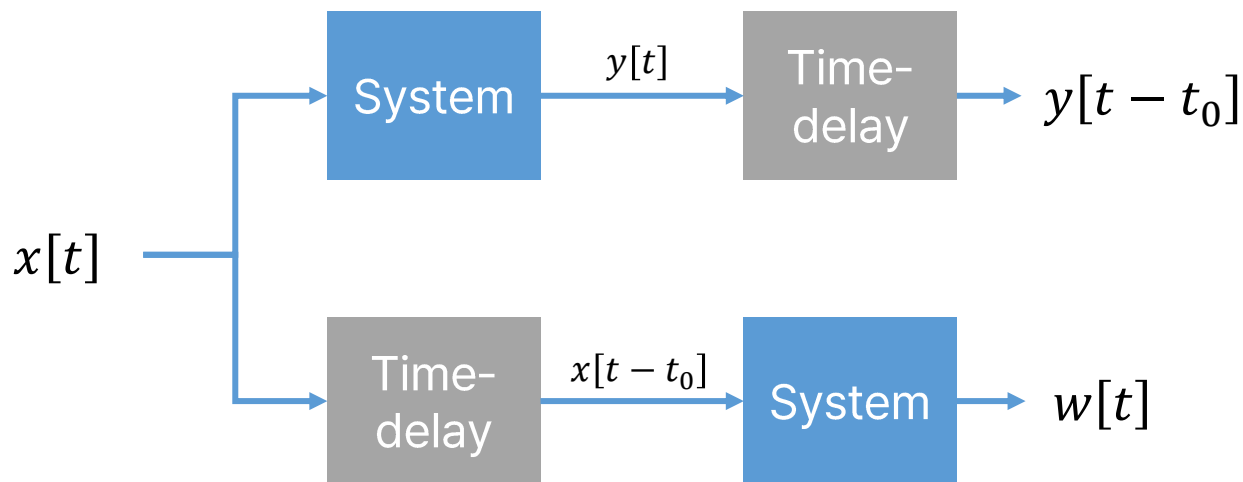


Review: Signals and Systems

- Linear, Time-invariant System

- **Time-invariant:** system does not change by time.
- $x[t] \rightarrow y[t]$ 일 때,

$$x[t - \tau] \rightarrow y[t - \tau]$$



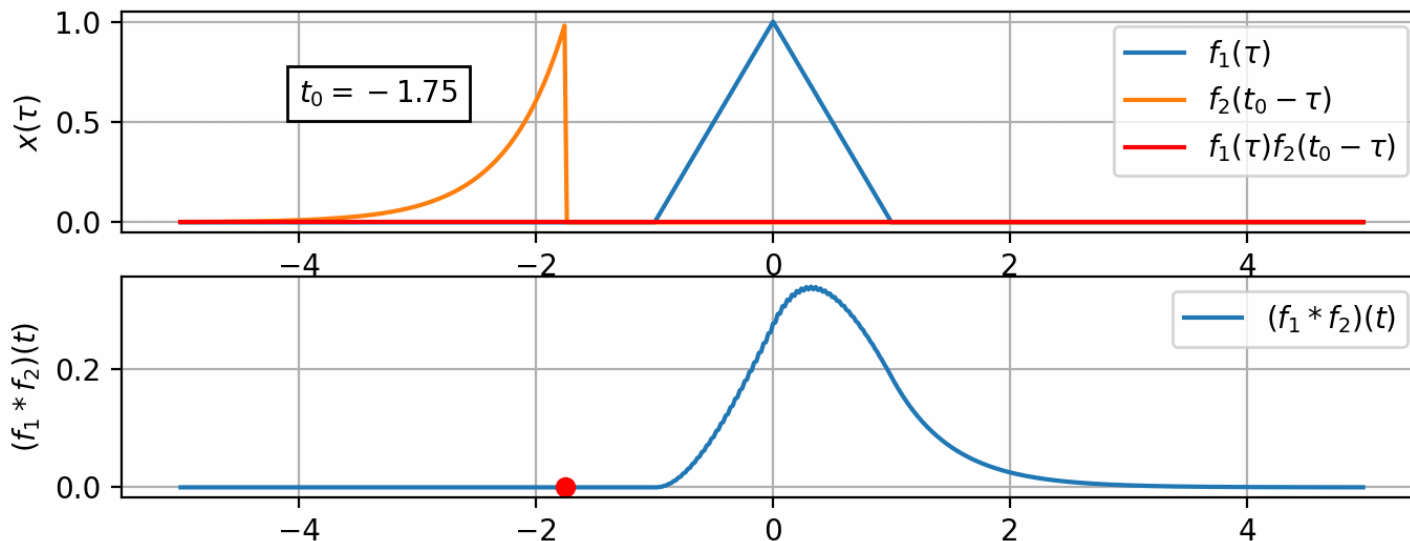
$$y[t - t_0] = w[t] \text{ for Time-invariant System}$$

Review: Signals and Systems

- Convolution

- A significant operation for **LTI** systems.
- Note that the function is time-invariant

$$x * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau.$$



Review: Signals and Systems

- Linear, Time-invariant System

signal $x(t) \Leftrightarrow$ **Weighted** Impulses

$$x(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau) d\tau$$



Apply the system to the input

$$\begin{aligned} y(t) &= H(x(t)) \\ &= H\left(\int_{-\infty}^{\infty} x(\tau) \delta_{\tau}(t) d\tau\right) \end{aligned}$$

If system is LTI
the order can be changed

$$y(t) = \int_{-\infty}^{\infty} x(\tau) H(\delta_{\tau}(t)) d\tau.$$

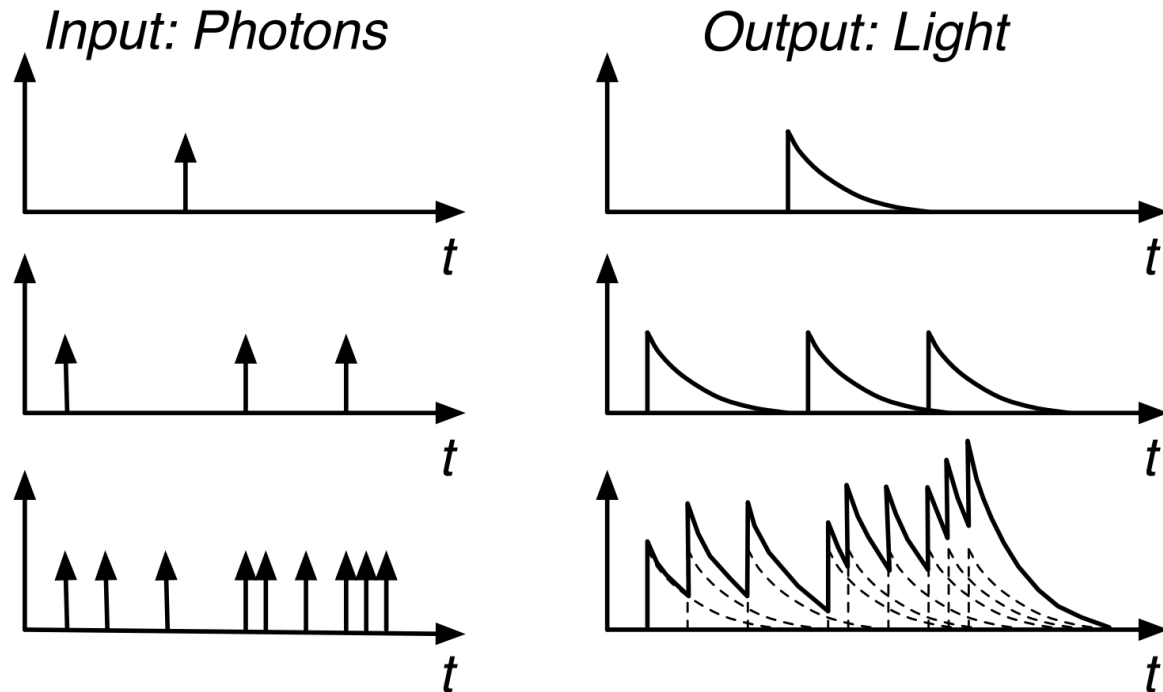
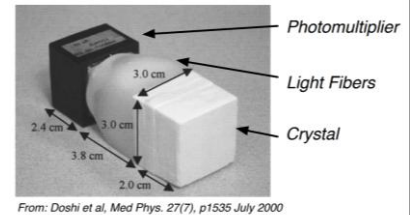


An LTI system is represented by
conv(input, Impulse Resp.)

$$y = x * h.$$

Review: Signals and Systems

- Linear, Time-invariant System
 - High energy photon detectors
 - Output is superposition of impulse responses



Review: Signals and Systems

- 선형대수학과의 연관성
 - Signal: a **vector** (or a matrix) (*Vector: a matrix with single column/row*)
 - System: a **function** of signal $\rightarrow$ maybe.. calculation of a matrix?
- Example)** $y[n] = x[n] - x[n - 1]$

	Output Signal	A System $\mathcal{H}$	Input Signal
	$N \times 1$	$N \times N$	$N \times 1$
$y[0] = x[0]$ $y[1] = x[1] - x[0]$ $y[2] = x[2] - x[1]$ $y[3] = x[3] - x[2]$...	$\rightarrow \begin{bmatrix} y[0] \\ y[1] \\ y[2] \\ y[3] \\ \vdots \\ y[n-1] \end{bmatrix}$	$= \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ \vdots \\ ? \end{bmatrix}$	$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ \vdots \\ x[n-1] \end{bmatrix}$

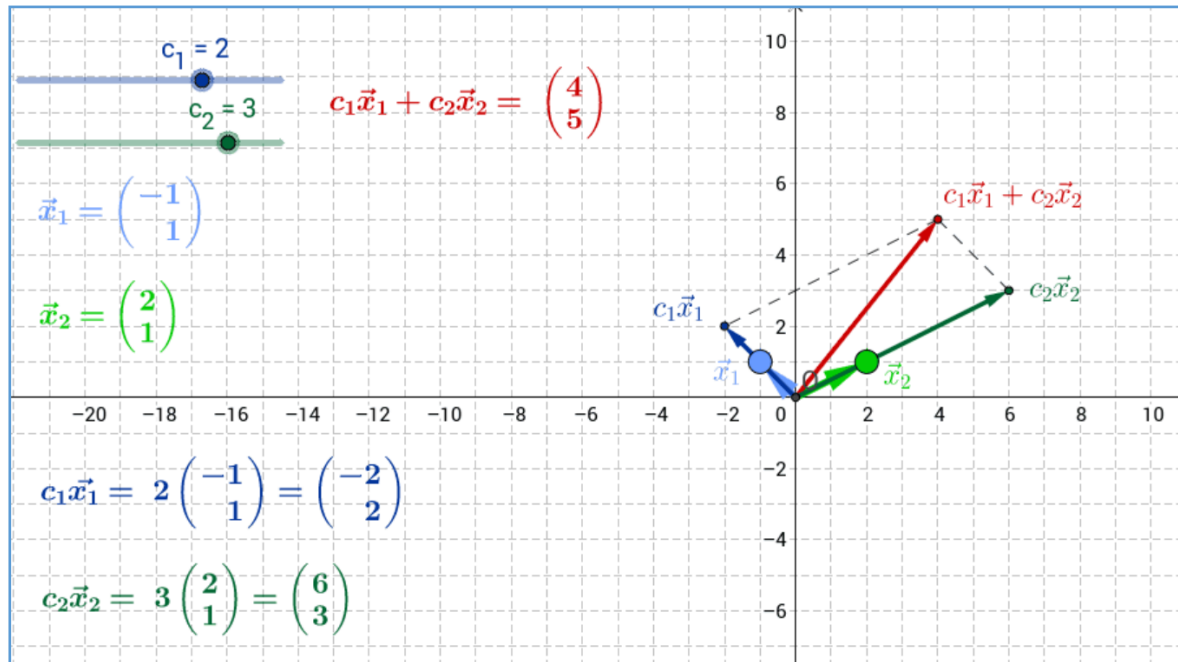
Review: Signals and Systems

- Linear, Time-invariant System

- Linearity:** scalar product, additivity (= **vector**)

- $x[n] \rightarrow y[n]$ 일 때,

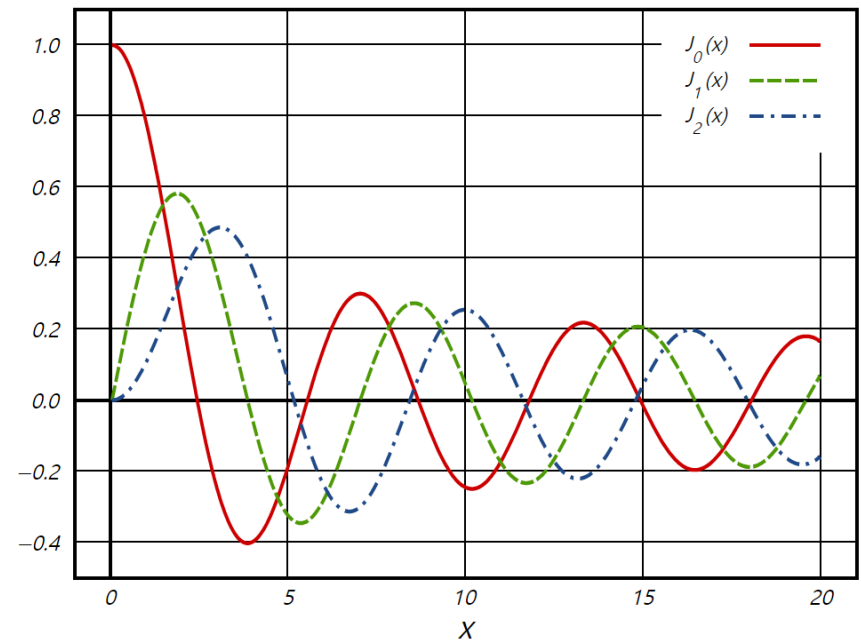
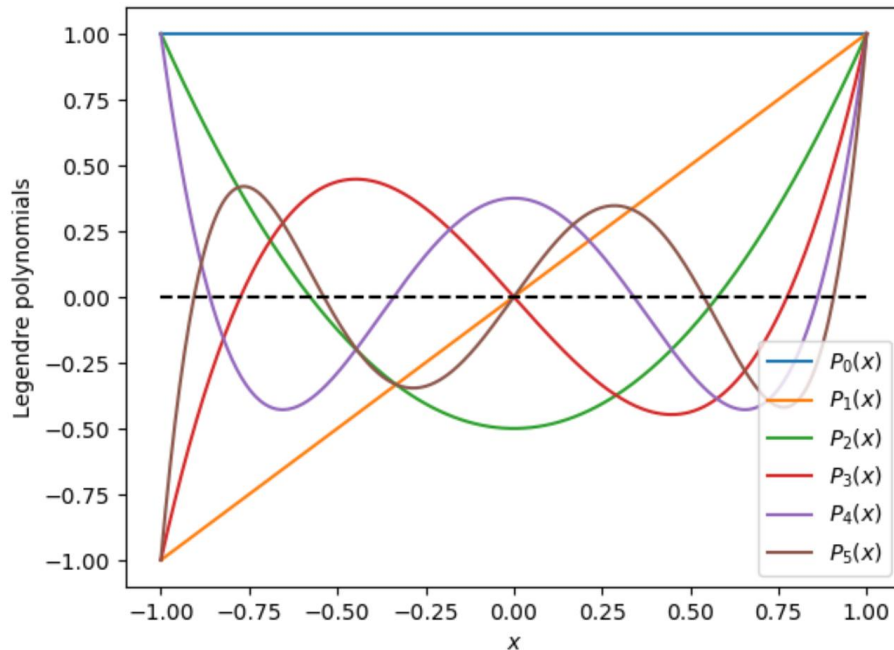
$$x[n] = \alpha x_1[n] + \beta x_2[n] \rightarrow y[n] = \alpha y_1[n] + \beta y_2[n]$$



Review: Signals and Systems

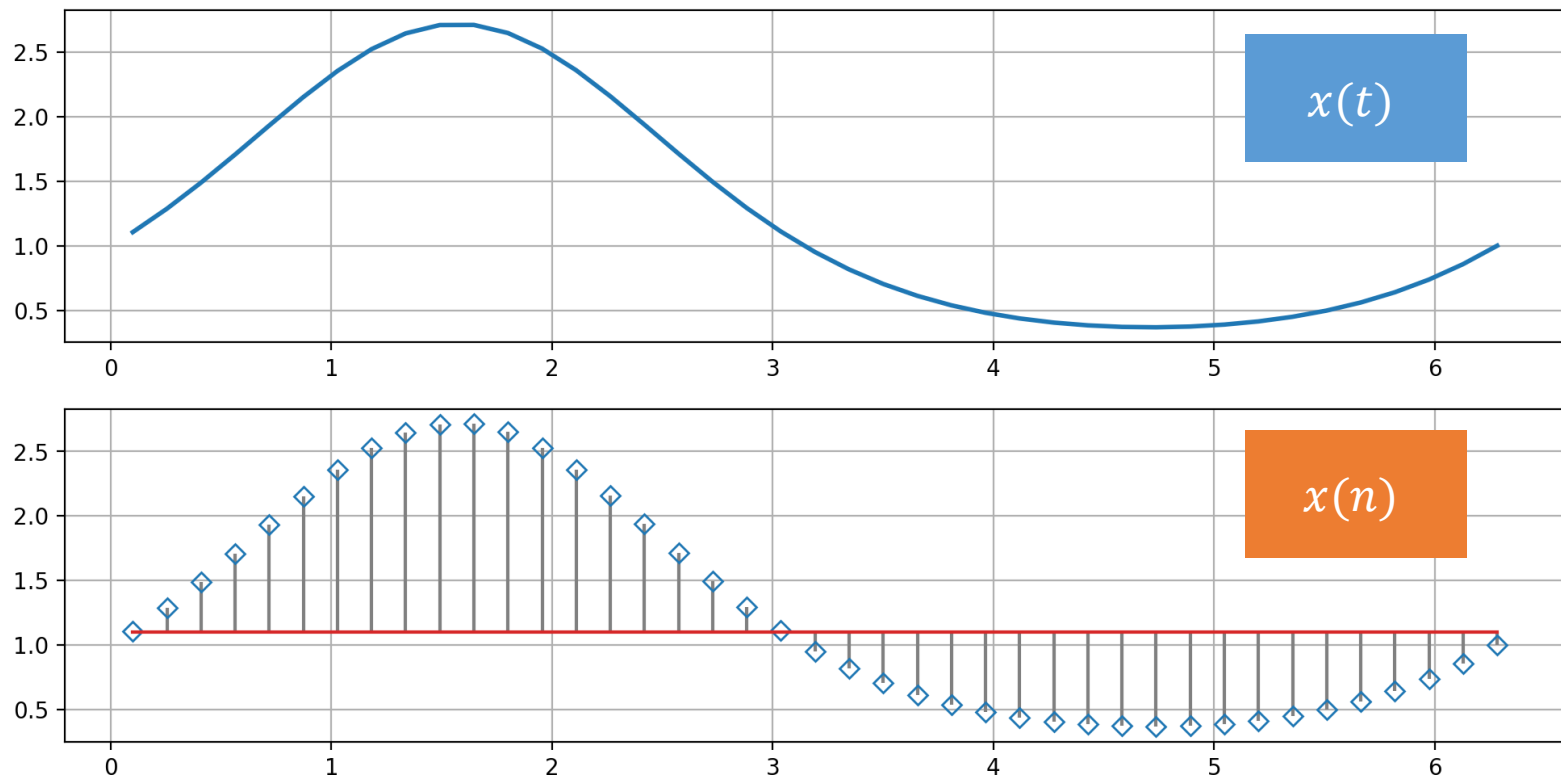
- Concept of a **Signal Space**

- A Vector represented by **some linear combination of basis**.
- **A signal can be represented by some orthogonal signals (Kernels).**
 - Key idea of "transformation"



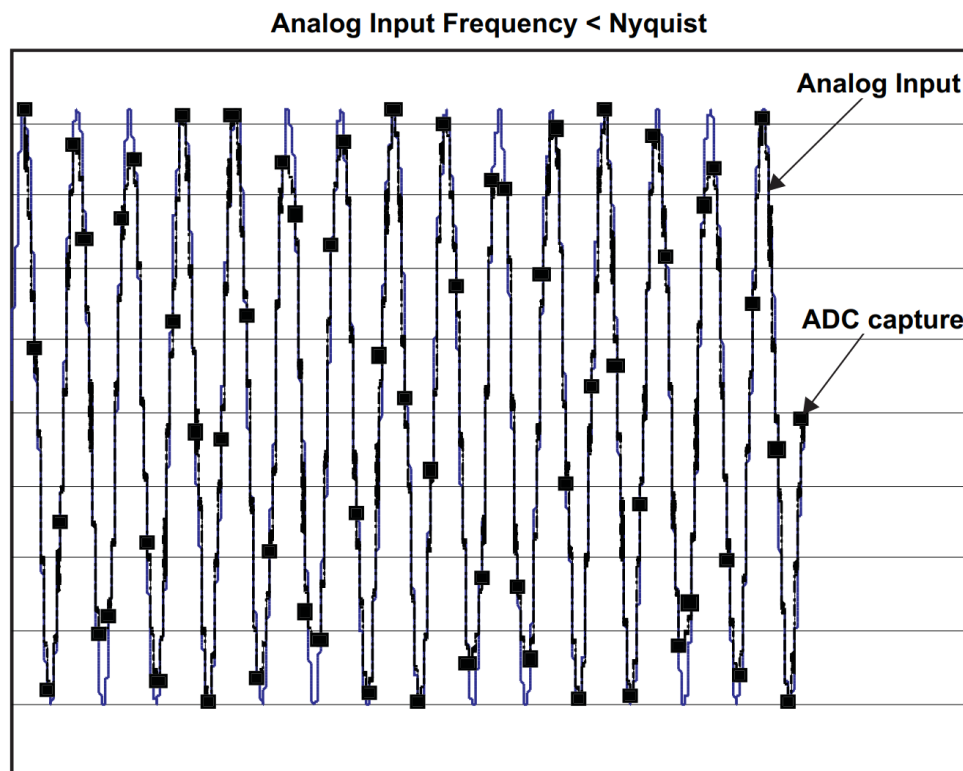
Review: Signals and Systems

- Sampling of a Continuous Signal
 - How is it possible?



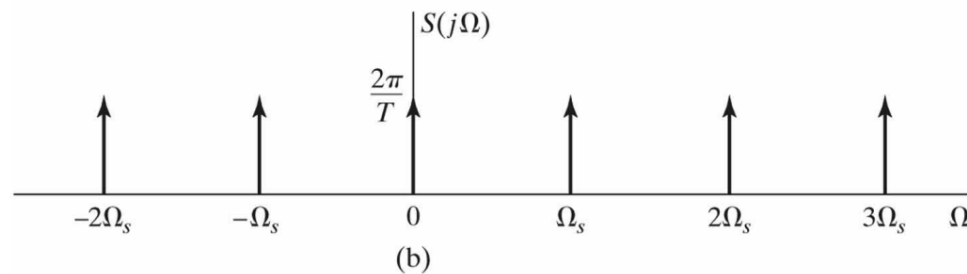
Review: Signals and Systems

- Sampling of a Continuous Signal
 - **Sampling = Signal * Impulse Train**



Review: Signals and Systems

- Sampling of a Continuous Signal
 - Consider an **Impulse Train (IT)**

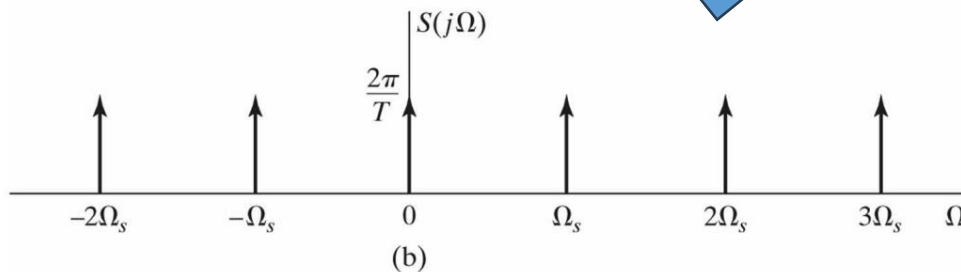
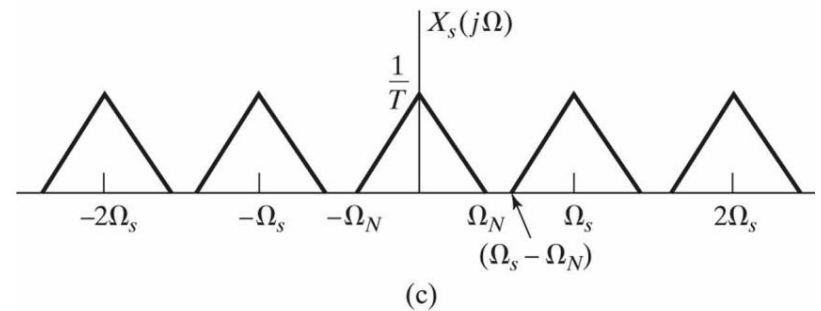
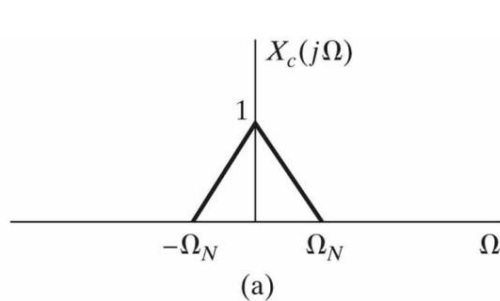
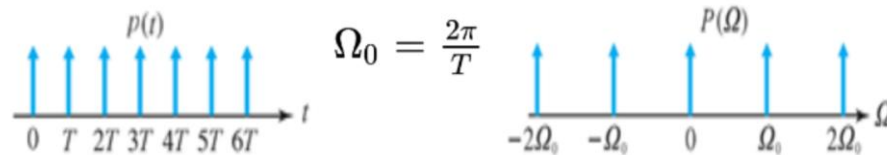


- Fourier Transform of IT is another IT. ($T \rightarrow 2\pi/T$)

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT) \quad \longleftrightarrow \quad F[x(t)] = \Omega_0 \sum_{k=-\infty}^{\infty} \delta(\Omega - k\Omega_0)$$

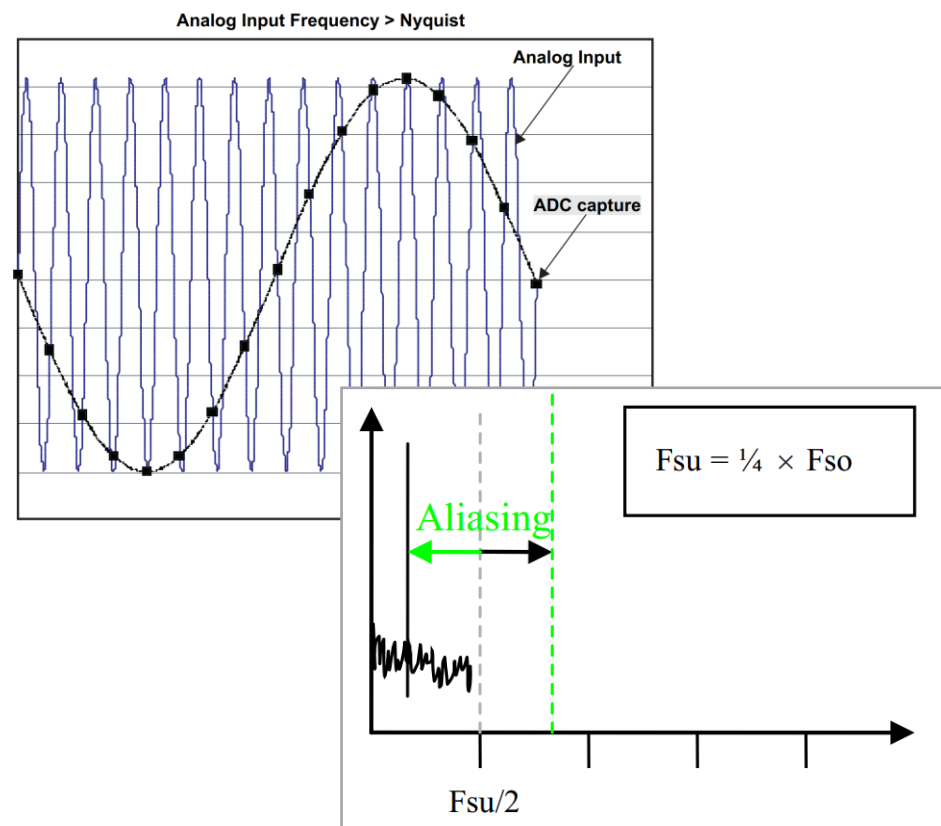
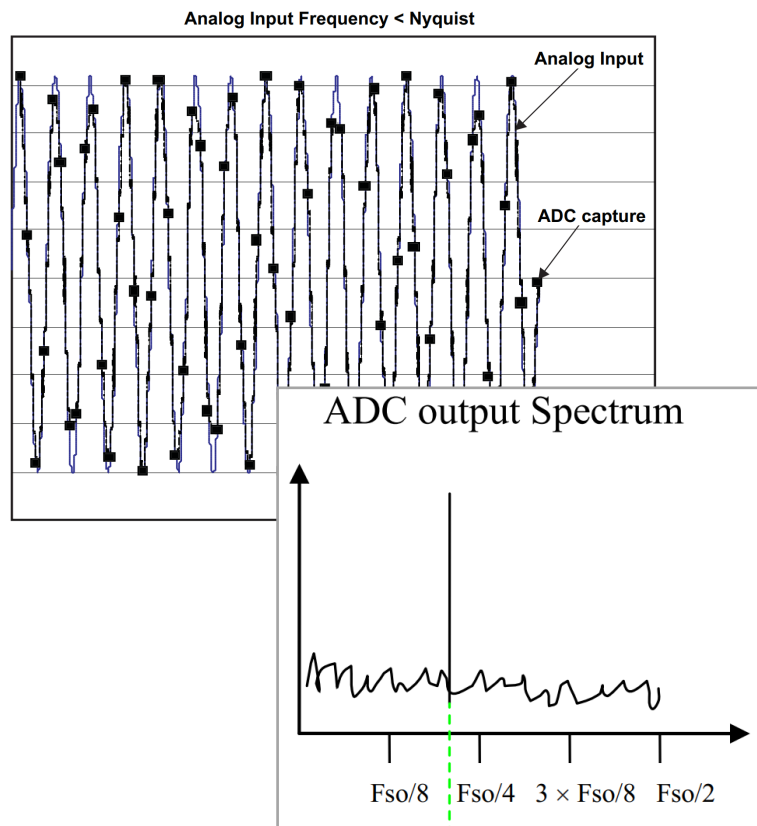
Review: Signals and Systems

- Sampling of a Continuous Signal
 - Recall that IT in Time domain == IT in Freq. Domain



Review: Signals and Systems

- Sampling of a Continuous Signal
 - Nyquist Criteria: $F_{in} < F_s/2$



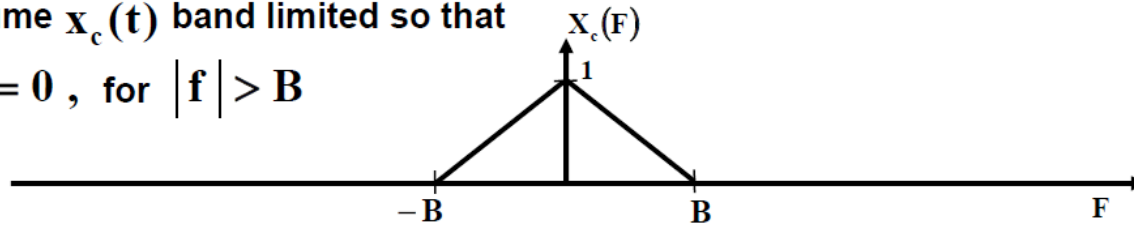
Review: Signals and Systems

- Sampling of a Continuous Signal

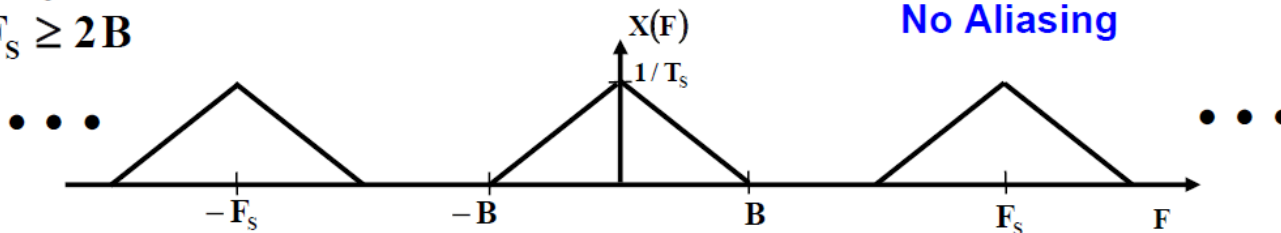
- Problem of a Sampling: Aliasing

- Assume $x_c(t)$ band limited so that

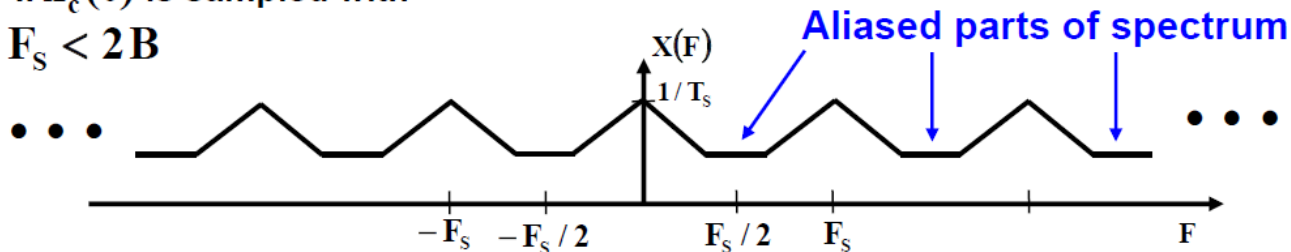
$$X(f) = 0, \text{ for } |f| > B$$



- If $x_c(t)$ is sampled with $F_s \geq 2B$

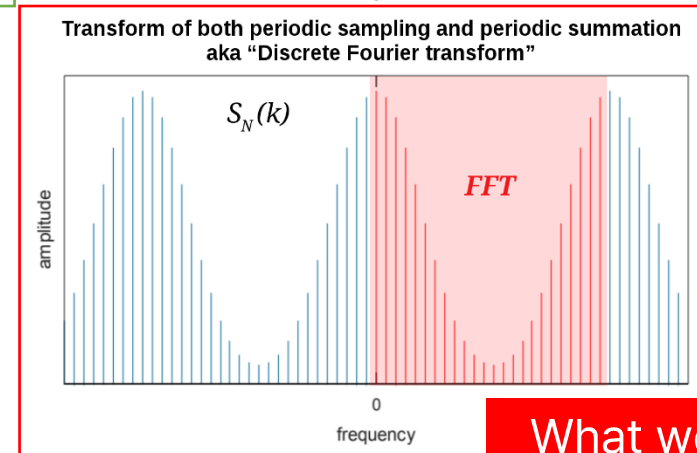
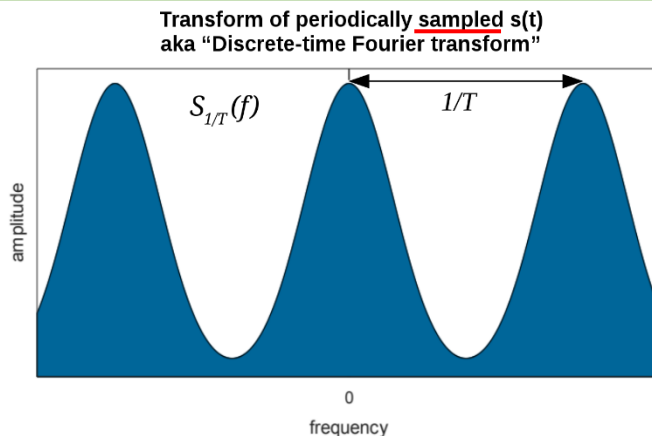
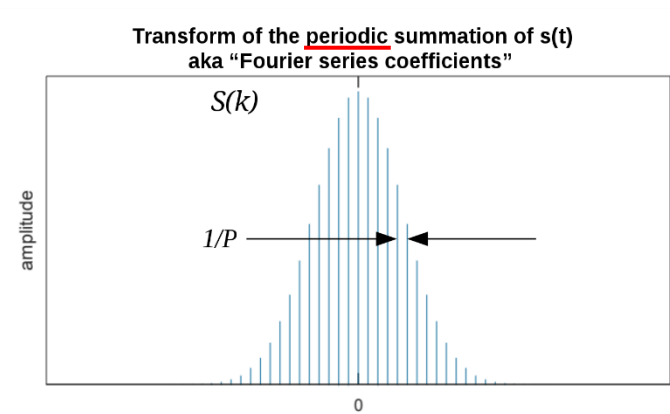
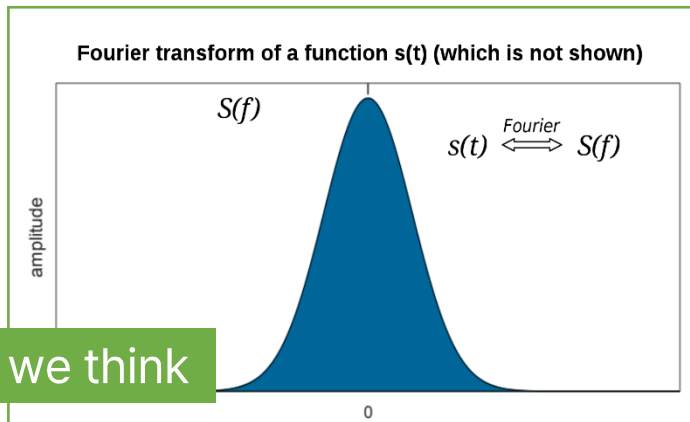


- If $x_c(t)$ is sampled with $F_s < 2B$



Review: Signals and Systems

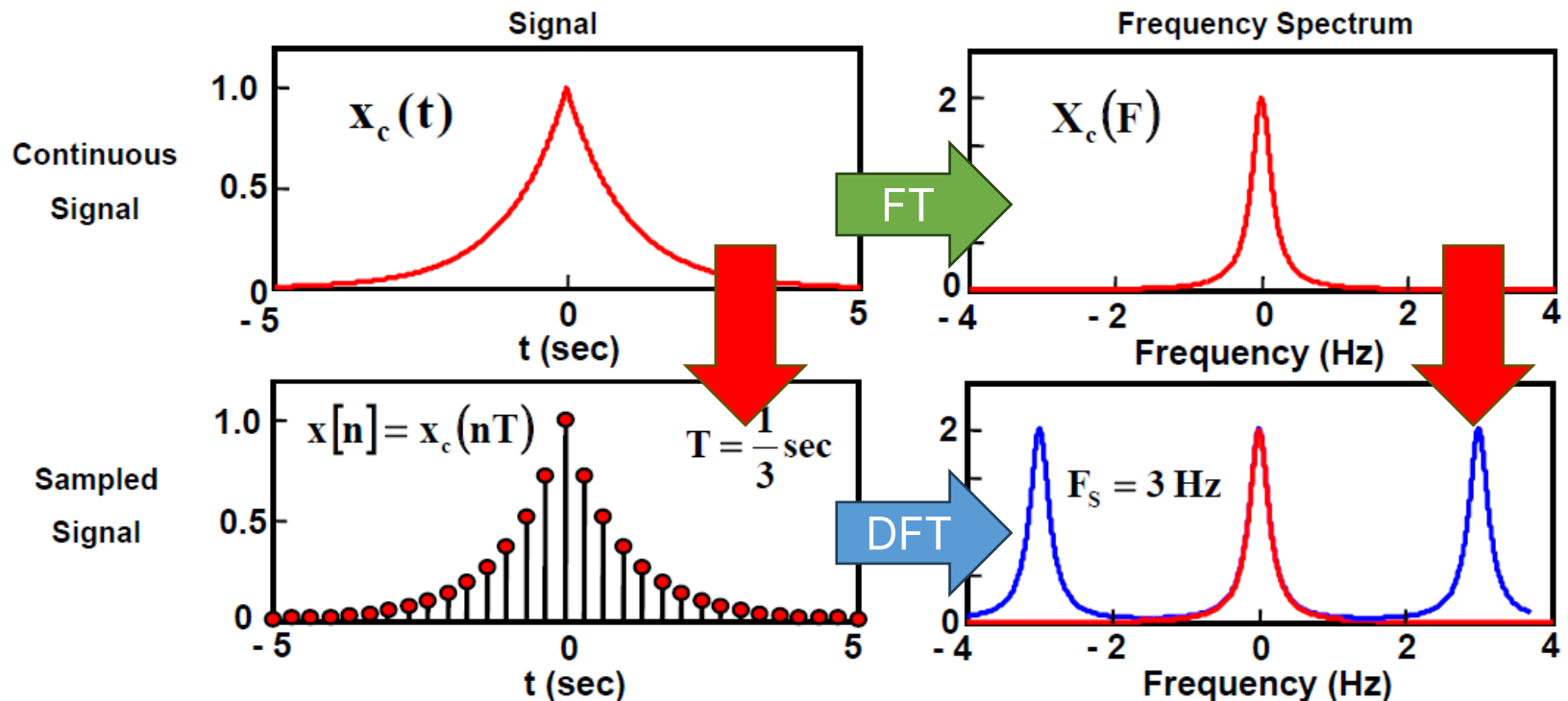
- Fourier Series & Transform
 - Understanding Sampling & Transform together



What we get

Fourier Analysis of Discrete-Time Systems

- Discrete Time Fourier Transform
 - Two systems applied in cascade way.
 - **The continuous Fourier Transform**
 - **The sampling process**



Fourier Analysis of Discrete-Time Systems

- Discrete Time Fourier Transform

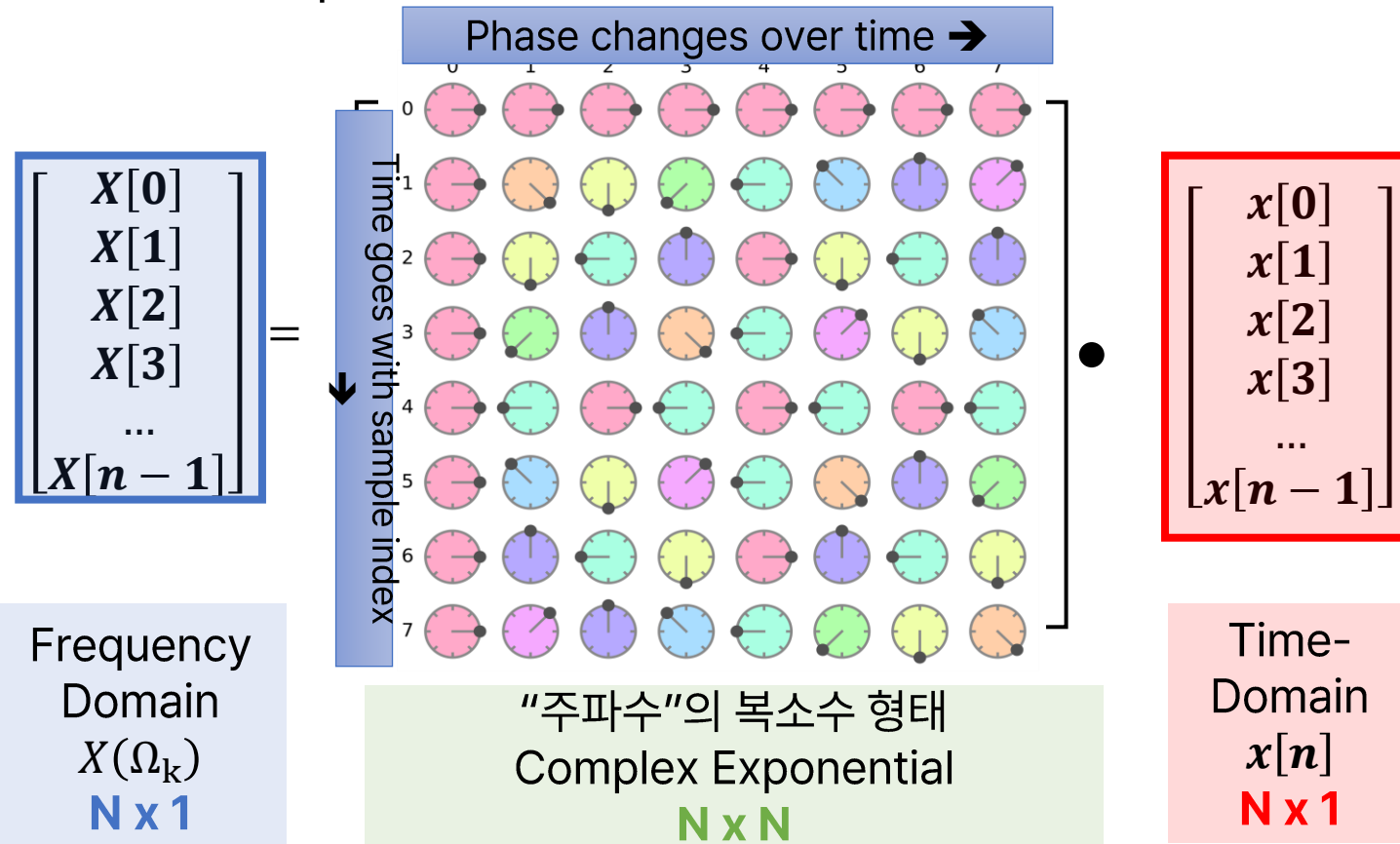
$$X(\Omega_k) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi k}{N}n}$$

- We have two indexes, **n** and **k**
 - N : discrete time index
 - K : discrete frequency index
 - Note that N is the number of samples of x(n)
- The transform can be performed by multiplication of a matrix!

Fourier Analysis of Discrete-Time Systems

- Discrete Fourier Transform
 - A visual explanation

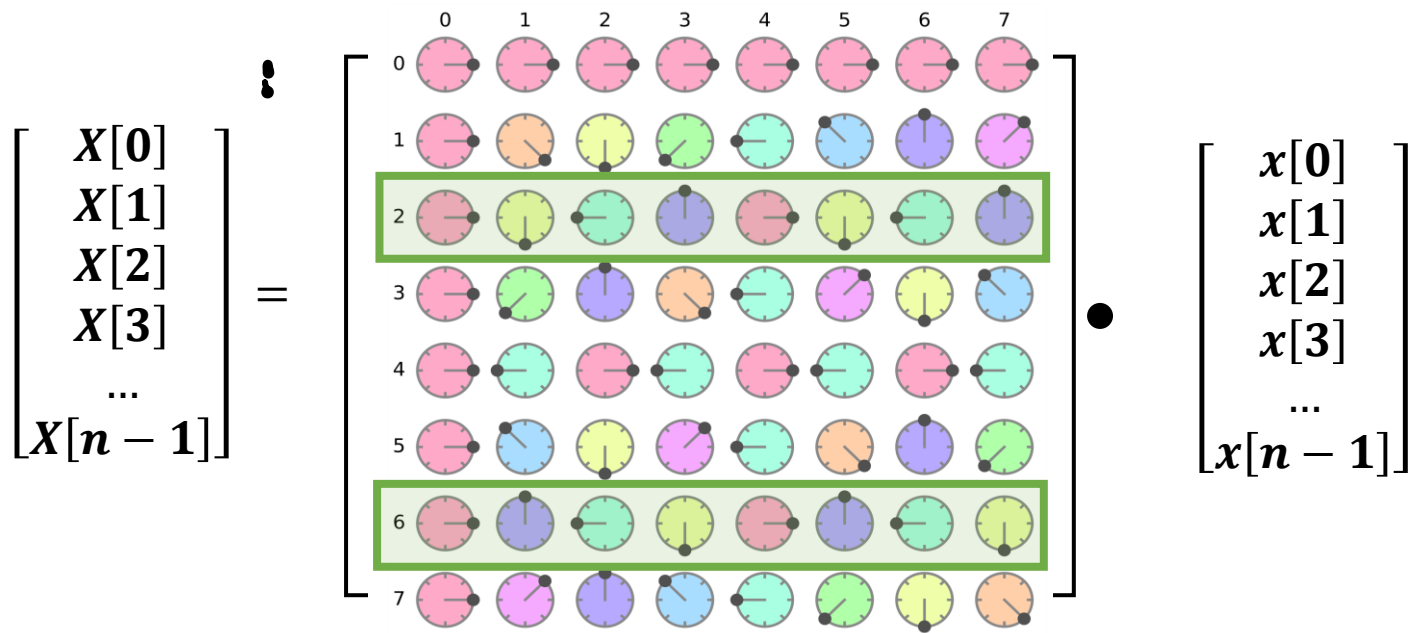
$$X(\Omega_k) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi k}{N} n}$$



Fourier Analysis of Discrete-Time Systems

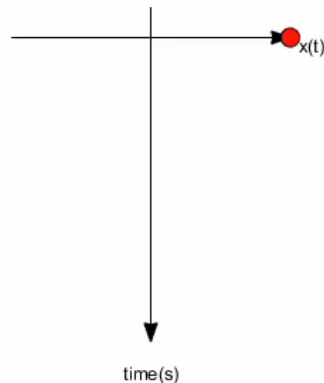
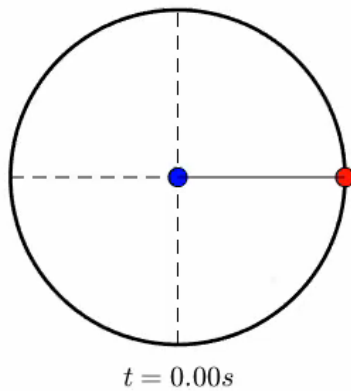
- Discrete Fourier Transform
 - The negative frequency
 - The symmetry of DFT of real signal

$$X(\Omega_k) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi k}{N} n}$$



Fourier Analysis of Discrete-Time Systems

- Discrete Fourier Transform
 - Why DFT result of complex signal is not symmetrical?



$$e^{j\omega t} = \cos \omega t + j \sin \omega t$$

Fourier Analysis of Discrete-Time Systems

- Properties of DFT

Property	Time Domain	Frequency Domain
Linearity	$a_1x_1(n) + a_2x_2(n)$	$a_1X_1(\Omega) + a_2X_2(\Omega)$
Translation	$x(n - n_0)$	$e^{-j\Omega n_0}X(\Omega)$
Modulation	$e^{j\Omega_0 n}x(n)$	$X(\Omega - \Omega_0)$
Conjugation	$x^*(n)$	$X^*(-\Omega)$
Time Reversal	$x(-n)$	$X(-\Omega)$
Upsampling	$(\uparrow M)x(n)$	$X(M\Omega)$
Downsampling	$(\downarrow M)x(n)$	$\frac{1}{M} \sum_{k=0}^{M-1} X\left(\frac{\Omega - 2\pi k}{M}\right)$
Convolution	$x_1 * x_2(n)$	$X_1(\Omega)X_2(\Omega)$
Multiplication	$x_1(n)x_2(n)$	$\frac{1}{2\pi} \int_{2\pi} X_1(\theta)X_2(\Omega - \theta)d\theta$
Freq.-Domain Diff.	$nx(n)$	$j \frac{d}{d\Omega} X(\Omega)$
Differencing	$x(n) - x(n - 1)$	$(1 - e^{-j\Omega})X(\Omega)$
Accumulation	$\sum_{k=-\infty}^n x(k)$	$\frac{e^{j\Omega}}{e^{j\Omega} - 1} X(\Omega) + \pi X(0) \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k)$

Fourier Analysis of Discrete-Time Systems

- Properties of DFT

Property	
Periodicity	$X(\Omega) = X(\Omega + 2\pi)$
Parseval's Relation	$\sum_{n=-\infty}^{\infty} x(n) ^2 = \frac{1}{2\pi} \int_{2\pi} X(\Omega) ^2 d\Omega$
Even Symmetry	$x \text{ is even} \Leftrightarrow X \text{ is even}$
Odd Symmetry	$x \text{ is odd} \Leftrightarrow X \text{ is odd}$
Real / Conjugate Symmetry	$x \text{ is real} \Leftrightarrow X \text{ is conjugate symmetric}$

Fourier Analysis of Discrete-Time Systems

- Fast Fourier Transform
 - The symmetry of DFT Matrix
 - Odd vs. Even Decomposition

$$X(\Omega_k) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi k}{N} n}$$

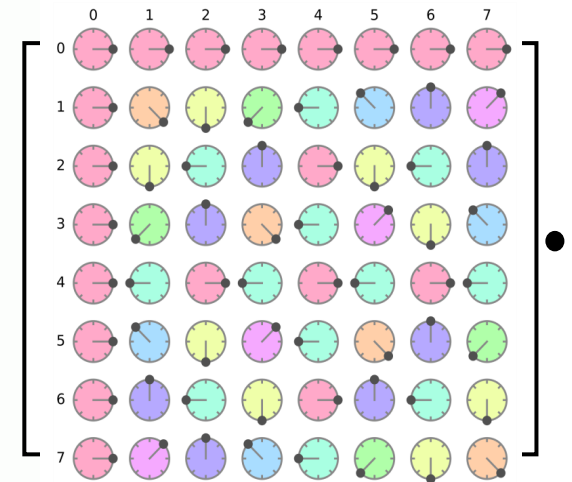
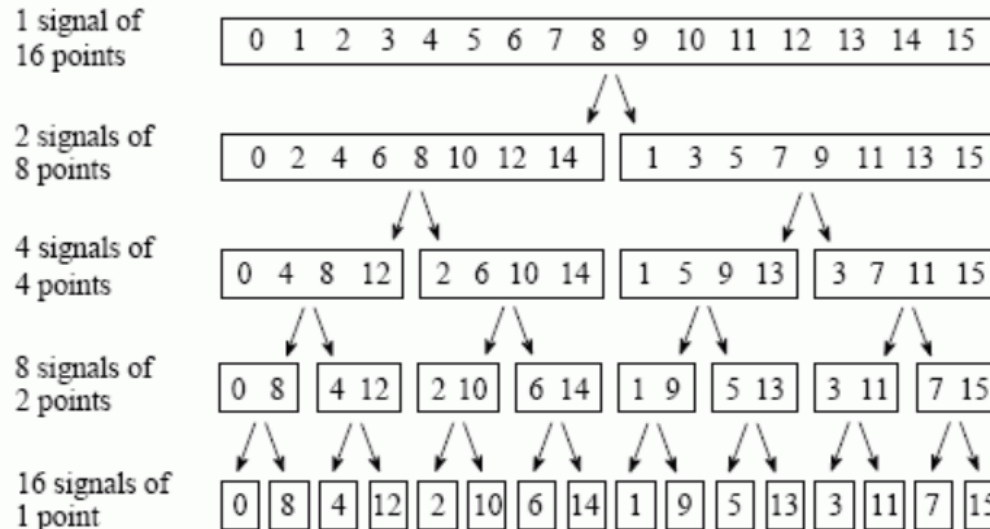


FIGURE 12-2

The FFT decomposition. An N point signal is decomposed into N signals each containing a single point. Each stage uses an *interleave decomposition*, separating the even and odd numbered samples.

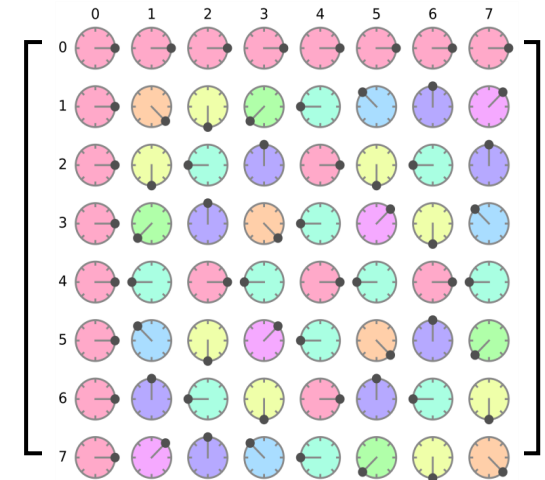
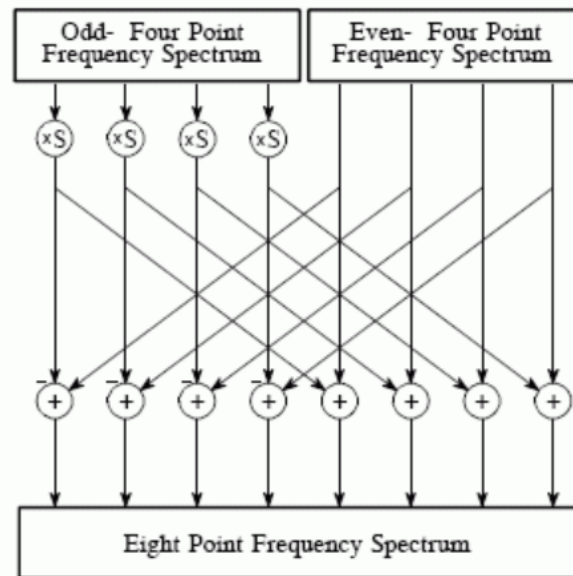
Fourier Analysis of Discrete-Time Systems

- Fast Fourier Transform
 - The symmetry of DFT Matrix
 - $O(n^2)$ vs. $O(n \log n)$

$$X(\Omega_k) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi k}{N} n}$$

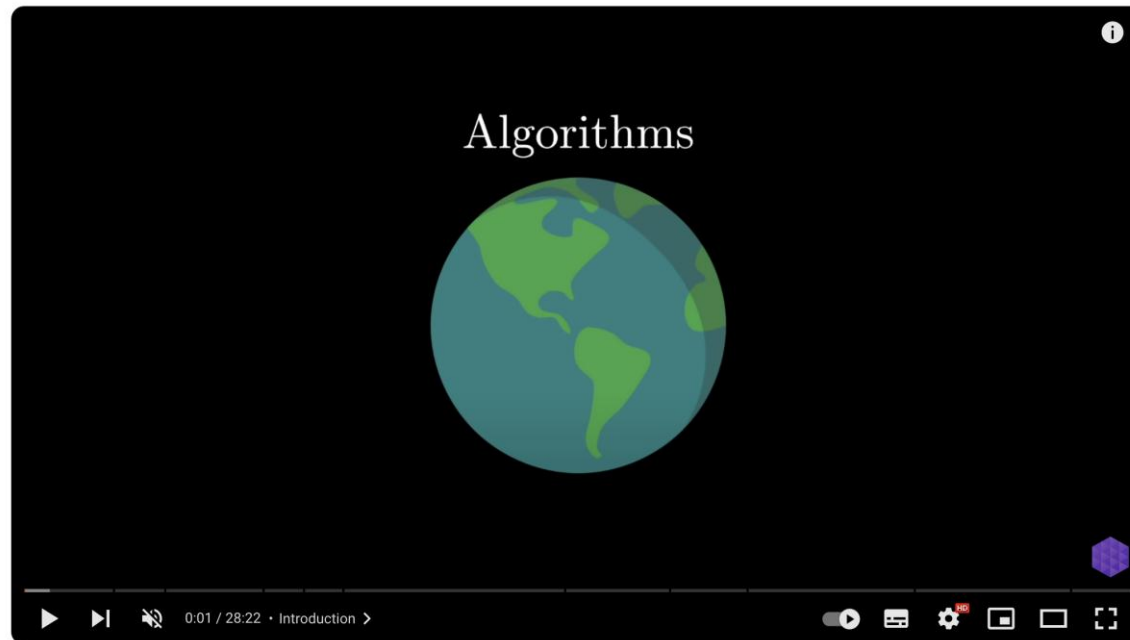
FIGURE 12-5

FFT synthesis flow diagram. This shows the method of combining two 4 point frequency spectra into a single 8 point frequency spectrum. The $\times S$ operation means that the signal is multiplied by a sinusoid with an appropriately selected frequency.



Fourier Analysis of Discrete-Time Systems

- Fast Fourier Transform (FFT)
 - Using the symmetry of odd and even function
 - The computational complexity, problem solving skill



The Fast Fourier Transform (FFT): Most Ingenious Algorithm Ever?



구독

5만



공유

오프라인 저장

Thanks



Thank you

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